

Proving Theorems About Lines & Angles

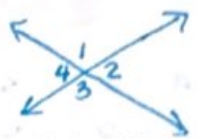
Objective: Students will critically think to come up with proofs about lines and angles.

Naming an Angle: How many different ways can you name this angle?

$\angle CBA$, $\angle ABC$, $\angle 1$, $\angle B$
 acute



Example 1: Construct two intersecting lines l and m , and label each angle as 1-4. What can you tell me about these angles?



$\angle 1 \cong \angle 3$
 $\angle 2 \cong \angle 4$
 $\angle 1 + \angle 2 = 180^\circ$
 $\angle 3 + \angle 4 = 180^\circ$
 $\angle 1 + \angle 4 = 180^\circ$
 $\angle 2 + \angle 3 = 180^\circ$

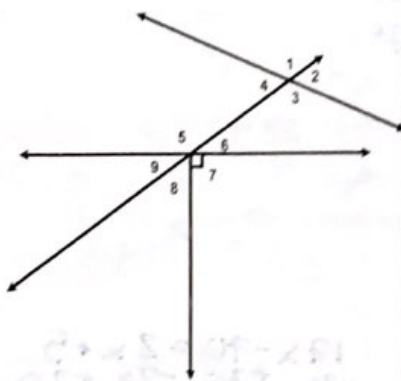
Example 2: Prove the Vertical Angle Theorem

Given: Lines l and m intersect to form vertical angles $\angle 1$ and $\angle 3$.
Prove: Vertical Angles Theorem: $m\angle 1 \cong m\angle 3$



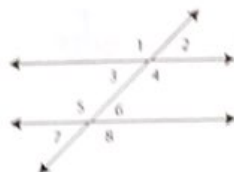
Statement	Reason
1. Lines l and m intersect to form VA. $\angle 1$ and $\angle 3$	1. Given
2. $\angle 1 + \angle 2 = 180^\circ$	2. Linear Pair
3. $\angle 2 + \angle 3 = 180^\circ$	3. Linear Pair
4. $\angle 1 + \angle 2 = \angle 2 + \angle 3$	4. Substitution Property
5. $\angle 1 = \angle 3$	5. Subtraction Property

Vocabulary: Different Kinds of Angles



Adjacent Angles: Angles "next to" each other.
Examples: $\angle 1$ and $\angle 2$, $\angle 5$ and $\angle 6$, $\angle 9$ and $\angle 8$
Linear Pairs: A pair of angles that form a line.
Examples: $\angle 1$ and $\angle 2$, $\angle 2$ and $\angle 3$, $\angle 5$ and $\angle 6$
Vertical Angles: Opposite angles made by two intersecting lines that are congruent (EQUAL).
Examples: $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$
Supplementary Angles: Angles that add up to 180°
Examples: $\angle 1$ and $\angle 4$, $\angle 3$ and $\angle 4$
Right Angles: 90° Angles
Examples: $\angle 9$ and $\angle 8$
Complementary Angles: Angles that add up to 90°
Examples: $\angle 9$ and $\angle 8$

More Vocabulary: Angles formed by two parallel lines cut by a transversal:



Example 3: Identify at least 2 pairs of the following and then determine if the angles are congruent or supplementary.

Linear Pairs	Vertical Angles	Corresponding Angles
$\angle 1$ and $\angle 2$ $\angle 7$ and $\angle 8$ $\angle 1$ and $\angle 3$ $\angle 5$ and $\angle 6$ $\angle 2$ and $\angle 4$...	$\angle 1$ and $\angle 4$ $\angle 2$ and $\angle 3$ $\angle 7$ and $\angle 6$	$\angle 2$ and $\angle 6$ $\angle 4$ and $\angle 8$ $\angle 1$ and $\angle 5$
Congruent OR Supplementary	Congruent OR Supplementary	Congruent OR Supplementary
Alternate Interior Angles	Alternate Exterior Angles	Same Side Interior
$\angle 3$ and $\angle 6$ $\angle 5$ and $\angle 4$	$\angle 1$ and $\angle 8$ $\angle 2$ and $\angle 7$	$\angle 4$ and $\angle 6$ $\angle 5$ and $\angle 3$
Congruent OR Supplementary	Congruent OR Supplementary	Congruent OR Supplementary
	Same Side Exterior	
	$\angle 2$ and $\angle 8$ $\angle 1$ and $\angle 7$	
	Congruent OR Supplementary	

Examples 4-5: Working with complementary and supplementary angles.

4. If $m\angle B = 37^\circ$, find the measure of its complement. 90°

$$m\angle A + 37^\circ = 90^\circ$$

$$\quad -37^\circ \quad -37^\circ$$

$$\boxed{m\angle A = 53^\circ}$$

5. $\angle A$ and $\angle B$ are supplementary. Find $m\angle A$ and $m\angle B$ if $m\angle A = 2x + 5$ and $m\angle B = 3x - 10$.

$$m\angle A + m\angle B = 180^\circ \quad m\angle A = 2(37^\circ) + 5$$

$$2x + 5 + 3x - 10 = 180^\circ \quad \boxed{m\angle A = 79^\circ}$$

$$5x - 5 = 180^\circ \quad m\angle B = 3(37^\circ) - 10$$

$$\quad +5 \quad +5 \quad \boxed{m\angle B = 101^\circ}$$

$$\frac{5x}{5} = \frac{185^\circ}{5}$$

$$x = 37^\circ$$

Examples 6-8: Given that the lines that appear parallel are parallel, solve for the variable(s):

6. alternate exterior

$$2x + 27 = 3x - 27$$

$$\quad +27 \quad +27$$

$$2x + 54 = 3x$$

$$\quad -2x \quad -2x$$

$$\boxed{54 = x}$$

7.

$$3x + 2x = 180^\circ$$

$$5x = 180^\circ$$

$$\boxed{x = 36}$$

$$y = 2(36)$$

$$\boxed{y = 72}$$

8.

$$17x - 70 = 2x + 5$$

$$\quad -2x \quad +70 \quad -2x \quad +70$$

$$15x = 75$$

$$\boxed{x = 5}$$

$$3y + 5 + 5y + 15 = 180^\circ$$

$$8y + 20 = 180^\circ$$

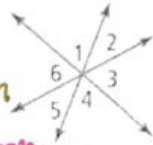
$$\quad -20 \quad -20$$

$$\frac{8y}{8} = \frac{160^\circ}{8} \quad \boxed{y = 20}$$

Example 9-10: Proof - Construct a two-columned proof for each of the following

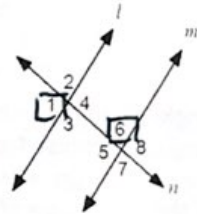
9. Given: $\angle 4 \cong \angle 6$
 Prove: $\angle 1 \cong \angle 3$

Statements	Reasons
1. $\angle 4 \cong \angle 6$	1. Given
2. $\angle 1 \cong \angle 4$	2. Vertical Angle Theorem
3. $\angle 3 \cong \angle 6$	3. Vertical Angle Theorem
4. $\angle 1 \cong \angle 3$	4. Substitution



10. Given that $l \parallel m$, prove that $\angle 1$ is supplementary to $\angle 6$

Statements	Reasons
1. $l \parallel m$	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$	2. Linear Pair
3. $\angle 2 \cong \angle 6$	3. Corresponding
4. $m\angle 1 + m\angle 6 = 180^\circ$	4. By substitution



Examples 11-14: Given that that $r \parallel s$ and $t \parallel u$, prove the following.

Statement	Reason
11. Prove that $\angle 7 \cong \angle 10$.	
1. $r \parallel s$ $t \parallel u$	1. Given
2. $\angle 7 \cong \angle 2$	2. Alt. Ext. Angles
3. $\angle 2 \cong \angle 10$	3. Corresponding
4. $\angle 7 \cong \angle 10$	4. Substitution

Statement	Reason
12. Prove that $\angle 5 \cong \angle 9$.	
1. $r \parallel s$ $t \parallel u$	1. Given
2. $\angle 5 \cong \angle 1$	2. Corresponding
3. $\angle 1 \cong \angle 9$	3. Corresponding
4. $\angle 5 \cong \angle 9$	4. Substitution

Statement	Reason
13. Prove that $\angle 7$ and $\angle 9$ are supplementary.	
1. $r \parallel s$ $t \parallel u$	1. Given
2. $\angle 7 + \angle 1 = 180^\circ$	2. Same side exterior
3. $\angle 1 \cong \angle 9$	3. Corresponding
4. $\angle 7 + \angle 9 = 180^\circ$	4. Substitution

Statement	Reason
14. Prove that $m\angle 5 + m\angle 10 = 180^\circ$	
1. $r \parallel s$ $t \parallel u$	1. Given
2. $\angle 5 + \angle 3 = 180^\circ$	2. Same side interior
3. $\angle 3 \cong \angle 10$	3. Alternate exterior
4. $m\angle 5 + m\angle 10 = 180^\circ$	4. substitution.

