

Right Triangle Similarity

Warm-up: Write down what you know about a right triangle: Draw one if your heart desires.

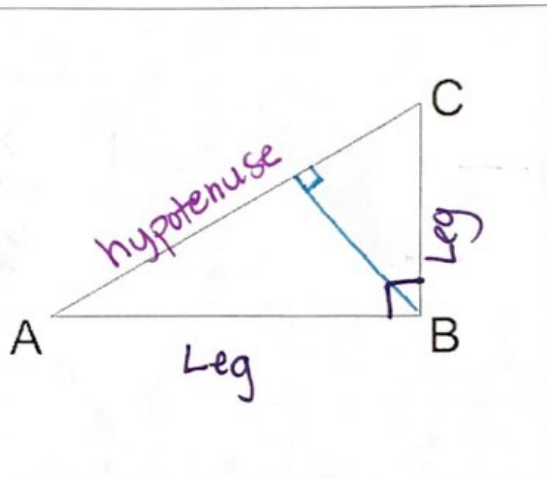
One 90° angle. $a^2 + b^2 = c^2$

Vocabulary:

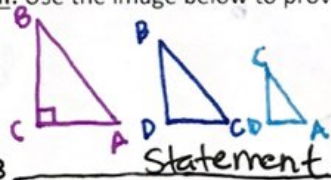
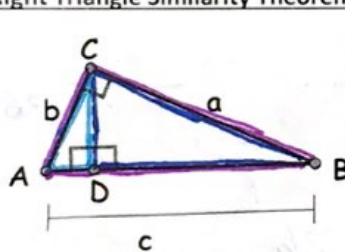
Hypotenuse - The longest side of a right triangle. Opposite the right \angle \overline{AC}

Legs - Two shorter sides of the triangle. \overline{AB} , \overline{BC}

Altitude - Line segment that passes through a vertex & meets the other side @ a 90° angle.



Right Triangle Similarity Theorem: Use the image below to prove that $\triangle ABC \sim \triangle ACD \sim \triangle CBD$



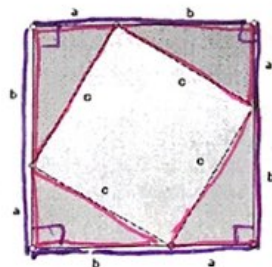
- | Statement | Reason |
|--|----------------------------|
| 1. $\angle ACB \cong \angle BDC \cong \angle ADC$ | 1. Right angles |
| 2. $\angle BAC \cong \angle DAC$ | 2. Reflective (same angle) |
| 3. $\triangle ABC \sim \triangle ACD$ | 3. AA~ |
| 4. $\angle ABC \cong \angle CBD$ | 4. Reflective |
| 5. $\triangle ABC \sim \triangle CBD$ | 5. AA~ |
| 6. $\triangle ABC \sim \triangle ACD \sim \triangle CBD$ | |

In other words:

- $\triangle ABC \sim \triangle ACD$
- $\triangle ABC \sim \triangle CBD$
- $\triangle ADC \sim \triangle CDB$

*You can use this idea to prove the Pythagorean Theorem, but you will do that in your homework.

Another proof of the Pythagorean Theorem:



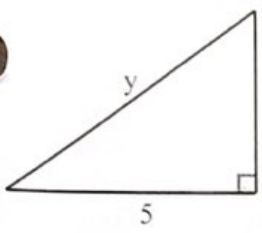
$$\begin{aligned} \text{Area: } & (a+b)^2 \\ &= (a+b)(a+b) \\ &= a^2 + 2ab + b^2 \end{aligned}$$

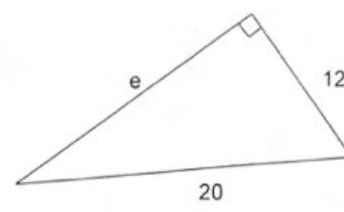
$$\begin{aligned} \text{Area: } & 4\left(\frac{1}{2}ab\right) + c^2 \\ &= 2ab + c^2 \end{aligned}$$

$$a^2 + 2ab + b^2 = 2ab + c^2$$

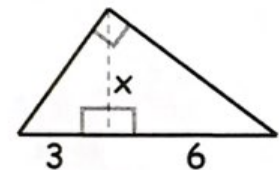
$$a^2 + b^2 = c^2$$

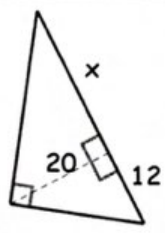
The Pythagorean Theorem: Find the unknown side length.

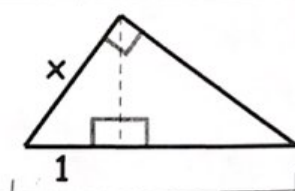
1.  $5^2 + 13^2 = y^2$
 $25 + 169 = y^2$
 $\sqrt{194} = \sqrt{y^2}$
 $y = 13.9$

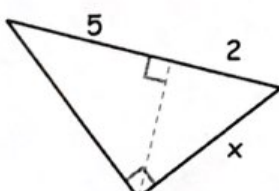
2.  $e^2 + 12^2 = 20^2$
 $e^2 + 144 = 400$
 $-144 \quad -144$
 $\sqrt{e^2} = \sqrt{256}$
 $e = 16$

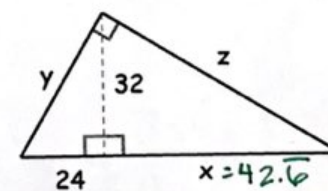
Examples using Right Triangle Similarity: Find the value of the variables in each right triangle. Write irrational values as simplified radicals AND as a decimal rounded to nearest tenth.

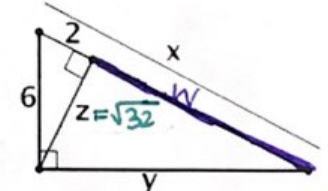
4.  $\frac{3}{x} = \frac{x}{6}$
 $\sqrt{18} = \sqrt{x^2}$
 $x = \sqrt{18} \approx 4.24$

5.  $\frac{20}{x} = \frac{12}{20}$
 $\frac{400}{12} = \frac{12x}{12}$
 $x = 33.\bar{3}$

6.  $\frac{1}{x} = \frac{x}{4}$
 $\sqrt{4} = \sqrt{x^2}$
 $x = 2$

7.  $\frac{2}{x} = \frac{x}{7}$
 $\sqrt{14} = \sqrt{x^2}$
 $x = \sqrt{14} \approx 3.74$

8.  $\frac{32}{24} = \frac{x}{32}$ $24^2 + 32^2 = y^2$
 $\frac{1024}{24} = \frac{24x}{24}$ $y = 40$
 $x = 42.\bar{6}$
 $32^2 + 42.\bar{6}^2 = z^2$
 $z = 53.\bar{33}$

9.  $z = \sqrt{32}$
 $z^2 + 2^2 = 6^2$ $\frac{z}{\sqrt{32}} = \frac{\sqrt{32}}{w}$
 $z^2 + 4 = 36$ $2w = 32$
 $-4 \quad -4$ $w = 16$
 $\sqrt{z^2} = \sqrt{32}$ $x = 18$
 $z = \sqrt{32}$
 $6^2 + y^2 = 18^2$
 $36 + y^2 = 324$
 $-36 \quad -36$
 $\sqrt{y^2} = \sqrt{288}$
 $y = 16.97$